Petri Net Models for Supply Chain Operational Management and Performance Evaluation

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Presentation Outline

- Introduction to Supply Chains (SC)
- Aim of the research
- SC operational model by First Order Hybrid Petri Nets (FOHPN)
- SC operational model by Coloured Timed Petri Nets (CTPN)
- Conclusions and Further Research
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Introduction to Supply Chains (SC)

Coordination center

Suppliers → Manufacturers → Distributors → Retailers → Customers → Recyclers

→ Material Flow
- Information Flow

**SC (Supply Chain):** collection of independent companies with complementary skills integrated with transportation and storage systems

**Aims of SC:**
- converting raw material into finished products
- improving profits
- decreasing the lead-time
- improve the customers service
SC definition: Operative level

- Entities of a SC:
  - 1 - Suppliers: provide raw material, components and semi-finished products.
  - 2 - Manufacturers/assemblers: transform input material/components into desired products.
  - 3 - Distributors: are intermediate nodes of materials flow.
  - 4 - Recyclers/de-manufacturers: they feed recovered material.
  - 5 - Customers or retailers: are sink nodes of material flow.
  - 6 - Logistics and transporters: connect the facilities.

The SC dynamics depends on the management methodologies specifying the business model, the information and material flow.

- **Make-to-Stock (MTS):** A push strategy so that customers are satisfied from stocks of inventory of finished goods.
- **(R, Q) inventory control rule for MTS:** fixed part quantities Q are ordered any time the stock level drops below the reorder level R.
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Aim of the research

Models based on PNs proposed in the related literature:


Limits: state space of the SC model is excessively large, the different types of products in the system are not modeled.


Limits: the paper does not face the basic aspect of the SC management and optimization problems at the operative level.

We propose two effective and efficient modular models for SC management and control at the operative level:

- 1) First Order Hybrid Petri Nets (FOHPN).
- 2) Coloured Timed Petri Nets (CTPN).
- The first may be used for testing /selection of SC operational control strategies.
- The second may be employed for performance evaluation and verification of the SC operation, enlightening the SC profitability, productivity, and promptness.
- Introduction to Supply Chains (SC)
- Aim of the research
- **SC operational model by First Order Hybrid Petri Nets (FOHPN)**
- **SC operational model by Coloured Timed Petri Nets (CTPN)**
- Conclusions and Further Research
A First Order Hybrid Petri Net Model for Supply Chain Management


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First-order hybrid Petri nets. An application to distributed manuf. systems


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M. Dotoli, Automatica.it, Pisa, 7-9 settembre 2011
FOHPN are introduced by


Key features of FOHPNs:

- **increasing in computational efficiency** with respect to place/transition models;
- **reducing the dimension of the state space** due to fluid approximation;
- **the possibility of using gradient information** to speed up optimization;
- **the possibility of modeling modular systems**.
We use a **hybrid model** to describe the dynamics of the SC that is traced by the flow of products between facilities and transporters.

The **continuous dynamics** models the flow of material, the manufacturing, the assembling of different products and the storage of products in the buffers.

The **discrete event dynamics** models the occurrence of stochastic events: the blocking of the raw material supply and transport operations due to strikes, shifts accidents, etc.
Model with FOHPN (II): Definition of PN

- A FOHPN is a bipartite digraph described by the seven-tuple $PN=(P, T, Pre, Post, Δ, F, RS)$.

- $P=P_c \cup P_d$ is the set of places that modeling resources:
  - places $P_c$ model flow of material (buffers and related capacity)
  - places $P_d$ model preferences, constraints and state (operative, failed) of SC entities

- $T=T_I \cup T_c \cup T_S \cup T_D$ transitions set
  - transitions $T_I$ models preferences
  - transitions $T_c$ models the continuous flow of material (production, assembling, recycling, load/unload operations)
  - transitions $T_S$ model the discrete flow of materials (transport, distribution), stochastics events (blocking, requests)
  - $T_D=\emptyset$
Model with FOHPN (III): Definition of PN

- Function $\Delta$ specifies the timing associated to timed transitions (zero for immediate transitions, exponential or triangular distribution for stochastic transitions, constant values for deterministic transitions).

- Function $F$ specifies $V_{mj}$ and $V_{Mj}$ the minimum and maximum firing speeds associated to $j$-th continuous transition.

- Function $RS$ associates a probability value called random switch to conflicting discrete transitions.

- Matrices $Pre$ and $Post$ describe the net structure.
A macro event occurs when:
- a discrete transition fires;
- a continuous place becomes empty;
- a continuous place, whose marking is increasing (decreasing), reaches a flow level that enables (disables) one or more discrete transitions.

The equation governing the dynamics of the discrete transitions is:

\[
m^c(\tau_k) = m^c(\tau^-_k) + C_{cd}\sigma(\tau_k)
\]

The equation governing the dynamics of the continuous transitions:

\[
\dot{m}_i(\tau) = \sum_{t_j \in T_c} C(p_i, t_j) v_j(\tau)
\]

Any admissible IFS (Instantaneous Firing Speeds, IFS) vector \( \mathbf{v} \) at \( m \) is a feasible solution of the following set of linear set \( S(PN, m) \):

\[
\begin{align*}
V_{\mathbf{v}_j} - v_j & \geq 0 \quad \forall t_j \in T_c(m) \\
v_j - V_{m_j} & \geq 0 \quad \forall t_j \in T_c(m) \\
v_j & = 0 \quad \forall t_j \in T_u(m) \\
\sum_{t_j \in T_c} C(p, t_j)v_j & \geq 0 \quad \forall p \in P_c(m)
\end{align*}
\]
Model with FOHPN (V):

The net dynamics: time driven and event driven

- The FOHPN is described by a discrete-time state equation:

\[ x(\tau_{k+1}) = A(\tau_k)x(\tau_k) + B(\tau_k)u(\tau_k) \]

- The behavior of the system is described in the macroperiod \([\tau_k, \tau_{k+1})\) by the following equations:

\[
\begin{bmatrix}
  m^c(\tau_{k+1}) \\
  m^d(\tau_{k+1}) \\
  v(\tau_{k+1})
\end{bmatrix} =
\begin{bmatrix}
  I & 0 & 0 \\
  0 & I & 0 \\
  0 & 0 & D(\tau_k)
\end{bmatrix}
\begin{bmatrix}
  m^c(\tau_k) \\
  m^d(\tau_k) \\
  v(\tau_k)
\end{bmatrix}
+ \begin{bmatrix}
  C_{cc}v(\tau_k) & C_{dc} \\
  0 & C_{dd}\sigma(\tau_k) \\
  f(\tau_k) & 0
\end{bmatrix}
\begin{bmatrix}
  \tau_{k+1} - \tau_k \\
  \sigma(\tau_{k+1})
\end{bmatrix}
\]

- The net state is given by the net marking and by the vector \(v(\tau_k)\) of timers associated to discrete timed transitions at time \(\tau_k\).

- The net input includes the macro-period duration and the firing vector \(\sigma(\tau_{k+1})\) associated to the firing of transition \(t_j\) at \(\tau_k\).

- The elements of matrix \(D(\tau_k)\) and vector \(\mathcal{A}(\tau_k)\) are equal to 0/1 and depend on the macro-event occurring at the sampling instant.

- The system output is:

\[ y(\tau_k) = E x(\tau_k) \]
Model with FOHPN (VI): Subsystems model of the SC

Input buffer module by the (R,Q) policy

$p_B$: the buffer of finite capacity
$p_B'$: the capacity $m_B + m_B' = C_B$
$t_i^T$: the trasport
$t_D^T$: the demand
$Q_i, Q_i'$: fixed order quantity
$R_B$: reorder level
Model with FOHPN (VII): Subsystems model of the SC

**Supplier module**

- $p_B$: the buffer of finite capacity
- $p'_B$: the capacity $m_B + m'_B = C_B$
- $t_1$: the arrival of material
- $p_k$: the supplier in operative state
- $p'_k$: the supplier blocked
- $t_k$: the blocking of material supply
- $t'_k$: the restoration of material supply
- $t_T$: transport operation

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Model with FOHPN (VIII):  
Subsystems model of the SC

Manufacturer/assembler module

$p_B$: the input buffer of finite capacity
$p_B'$: available capacity $m_B + m_B' = C_B$
$p_B$: the output buffer of finite capacity
$p_B'$: available capacity $m_B + m_B' = C_B$
$t_j$: production

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**Model with FOHPN (IX): Subsystems model of the SC**

**Transport module**

- $p_{Bi}$: downstream facility’s input buffer
- $p’_{Bi}$: downstream facility’s available capacity $m_{Bi} + m’_{Bi} = C_{Bi}$
- $p_k$: transporter in the operative state
- $p’_k$: transporter in the blocking state
- $t_k$: transporter blocking
- $t’_k$: transporter becomes operative
- $p_{Ci}$: control places that determines the choice of material to transport
- $p_{iT}$: it indicates if the transporter is available or not available
- $t^{T_i}$: transporters of $i$ items, triangular distribution
**Model with FOHPN (X): Subsystems model of the SC**

**Distributor Module**

\[ p_B: \text{buffer of finite capacity} \]

\[ p'_B: \text{the capacity } m_B + m'_B = C_B \]

\[ t^{T_i}: \text{transporter of } i \text{ items} \]

\[ t^{D_i}: \text{request of } i \text{ items} \]

\[ P_c: \text{control place} \]
Model with FOHPN (XI): Subsystems model of the SC

Retailer module

- \( p_B \): the buffer of finite capacity
- \( p'_B \): the capacity \( m_g + m'_B = C_B \)
- \( t_1 \): product request, exponential distribution
- \( p_F \): retailer
- \( t_L \): destruction of finished products
- \( p_S \): products to be de-manufactured
- \( p_D \): goods to be discarded
- \( t_T \): transport operation
De-manufacturer module

$p_{Bi}$: the input buffer of finite capacity
$p'_{Bi}$: the input buffer capacity $m_{Bi} + m'_{Bi} = C_{Bi}$
$p_{Bi}$: the output buffer of finite capacity
$p'_{Bi}$: the output buffer capacity $m_{Bi} + m'_{Bi} = C_{Bi}$
t$_j$: disassembly
Model with FOHPN (XIII): SC control

\[ x(\tau_{k+1}) = A(\tau_k)x(\tau_k) + B(\tau_k)u(\tau_k) \]

- \( A(\tau_k) \) and \( B(\tau_k) \) describe the SC in \([\tau_k, \tau_{k+1})\)
  and depend on the event occurring in \( t=\tau_k \)

- The vector IFS \( v(\tau_k) \in S(PN,m(\tau_k)) \) affects the matrix \( B(\tau_k) \)
  \( \Rightarrow \) Choice of \( v(\tau_k) \) in each macro-period on the basis of the knowledge of
  the SC state in order to optimize a particular objective function
  with \( y(\tau_k) = E \) and with \( m_r \) a reference vector

- Input vector \( u(\tau_k) \) determined by a decision maker

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Model with FOHPN (XIV):
SC control

- **Controller 1 (C1): flow maximization.**
  \[ m_r = 0, \ y = m^c, \ \max_y J_1 = \max_y (1^T \cdot y) \ \text{s.t.} \ v \in S(PN, m) \]

- **Controller 2 (C2): stored materials minimization.**
  \[ m_r = 0, \ y, \ \min_y J_2 \ \text{s.t.} \ v \in S(PN, m) \]

- **Controller 3 (C3): buffer inventory control.**
  \[ m_{ri} \ \text{reference value,} \ y, \\]
  \[ \min_y J_3 \ \text{s.t.} \ v \in S(PN, m) \]
3 suppliers

2 manufacturers

1 distributor

2 retailers

1 de-manufacturer

1 product (PC)

4 semi-finished products (M, K, C, H)

Case Study A (I): Desktop Computer production

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Case Study A (II): FOHPN model

3 suppliers
2 manufacturers
1 distributor
2 retailers
1 de-manufacturer

The model selects the production rates by solving the controller optimization problem.
Case Study A (III):
Performance indices and Controller

- **T = Throughput** average number of products obtained in a time unit
- **SI = System Inventory** average number of products stored in all the system buffers during the run time TP
- **LT = SI/T = Lead Time** a measure of the time spent by the SC to convert raw material into final products

- Controller C1 (flow maximization)
- Controller C2 (stored materials minimization)
  \[ P_y = \{ p_1, p_3, \ldots, p_{13}, p_{31}, p_{33} \} \]
- Controller C3-1 (buffer inventory control)
  \[ P_y = \{ p_{31}, p_{33} \}, m_{r31} = m_{r33} = 80 \]
- Controller C3-2 (buffer inventory control)
  \[ P_y = \{ p_1, p_3, \ldots, p_{13}, p_{31}, p_{33} \}, m_r = 60, i = 1, 3, 5, \ldots, 13, m_{r31} = m_{r33} = 80 \]
Case Study A (IV): Data

Buffers and transporters capacity, reorder level and fixed order quantities

<table>
<thead>
<tr>
<th>Capacities</th>
<th>Reorder levels</th>
<th>Fixed order quantities</th>
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</thead>
<tbody>
<tr>
<td>C_1, C_5, C_{17}, C_{15}</td>
<td>R_1 = 18</td>
<td>Q_1 = 50</td>
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<td>Q_2 = 45</td>
</tr>
<tr>
<td>C_{17}</td>
<td>R_3 = 25</td>
<td>Q_3 = 55</td>
</tr>
<tr>
<td>C_{37}, C_{19}</td>
<td>R_4 = 25</td>
<td>Q_4 = 40</td>
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<tr>
<td>C_{8}, C_{6}, C_{13}</td>
<td>R_5 = 15</td>
<td>Q_5 = 15</td>
</tr>
<tr>
<td>C_{7}, C_{27}</td>
<td>R_6 = 15</td>
<td>Q_6 = 15</td>
</tr>
<tr>
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<td>R_7 = 20</td>
<td>Q_7 = 30</td>
</tr>
<tr>
<td>C_{33}</td>
<td>R_8 = 20</td>
<td>Q_8 = 25</td>
</tr>
<tr>
<td>C_{31}</td>
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</tr>
<tr>
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<td>R_{10} = 10</td>
<td>Q_{10} = 5</td>
</tr>
<tr>
<td>C_{95}, C_{96}, C_{97}</td>
<td>R_{11} = 10</td>
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</tr>
<tr>
<td></td>
<td>R_{12} = 10</td>
<td>Q_{12} = 45</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Q_{13} = 40</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Q_{14} = 40</td>
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<tr>
<td></td>
<td></td>
<td>Q_{15} = 25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Q_{16} = 35</td>
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</table>

Continuous transitions

<table>
<thead>
<tr>
<th>[V_{min}, V_{max}]</th>
<th>Exponential</th>
<th>Average firing delay [hours]</th>
<th>Triangular</th>
<th>Average firing delay [hours]</th>
</tr>
</thead>
<tbody>
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<td>t_1, t_5, t_7</td>
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<td>t_{33}</td>
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<tr>
<td>t_2, t_3, t_4</td>
<td>3</td>
<td>t_{42}, t_{43}, t_{49}</td>
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<tr>
<td>t_6</td>
<td>4</td>
<td>t_{47}, t_{48}, t_{47}</td>
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</tr>
<tr>
<td>t_8</td>
<td>4</td>
<td>t_{54}, t_{57}, t_{59}</td>
<td>2</td>
<td></td>
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<tr>
<td>t_9</td>
<td>4</td>
<td>t_{64}, t_{65}, t_{62}</td>
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</tr>
<tr>
<td>t_{25}</td>
<td>5</td>
<td>t_{68}, t_{70}, t_{68}, t_{65}</td>
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<td>t_{69}, t_{70}, t_{69}</td>
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<td>t_{75}, t_{76}, t_{75}</td>
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<td>t_{77}, t_{78}, t_{77}</td>
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<td></td>
</tr>
<tr>
<td>t_{11}, t_{25}, t_{33}, t_{35}</td>
<td>21</td>
<td>t_{79}, t_{80}, t_{79}</td>
<td>22</td>
<td></td>
</tr>
</tbody>
</table>

Transitions continuous (discrete) firing speed (average time)
Case Study A (V): Simulation

- The model is implemented and simulated in MATLAB environment.

- At the beginning of each macro-period \( t = \tau_k \) the program describes and solves the optimization problem on the basis of the knowledge of \( x(\tau_k) \) and fixes \( v(\tau_k), A(\tau_k), B(\tau_k), \tau_{k+1} \) (next macro-event). After this it determines the next state \( x(\tau_{k+1}) \) by the state equation and the procedure is re-started.

- 50 independent replications of TP=600 h, with a transient period of TR=100 h. We obtain a half width that in the worst case equals 4.7% for a 95% confidence interval.
Case Study A (VI): Simulation results

- **Throughput** max with C1, min with C2, height with C3-1 e C3-2

- **System Inventory** min with C1, max with C1 and intermediate with C3-1 e C3-2 (less than C3-2)
Case Study A (VII): Simulation results

- **Lead time** min with C3-2.
- **Lead time** higher with C3-1 that with C3-2

- Buffer marking for final products $p_{31}$ into manufacturer M1.
- C1: buffer always full
- C2: buffer always empty
- C3-1 and C3-2 tend to keep the stock levels around the imposed set-point (80 items).
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A Coloured Timed Petri Net Model for Supply Chain Management and Performance Evaluation


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M. Dotoli, Automatica.it, Pisa, 7-9 settembre 2011
Compact and resource oriented model to describe the SC dynamics by the flow of batches of products among SC facilities by transporters.

The discrete event dynamics models stochastic events: client demands, block/restart of supplies, block/restart of transportations due to strikes, accidents.

The token color defines the product characteristics (e.g., type, quantity)

Advantages:

- Modularity
- Model suitable for simulation
- Applicability of control policies
- Possibility to distinguish the SC products: different dimensions and quantities, different manufacturing/transportation paths
A bipartite digraph described by the 9-tuple $\text{CTPN} = (P, T, Co, Pre, Post, \Omega, F, RS, M_0)$.

- $P$ set of places modelling activities: transportation, storage in manufacturers, buffers availability, presence of orders

- $T = T_F \cup T_I$ set of transitions where
  - Transitions in $T_F$ model the flow of jobs
  - Transitions in $T_I$ model the input of jobs in the system
• $CTPN = (P, T, Co, Pre, Post, \Omega, F, RS, MO)$

Coloured tokens represent jobs and orders. $\langle \lambda j, \theta j \rangle$

• Each token represents a batch of parts or of orders (raw material, semi-finished parts).

• $Co$ maps each place $p \in P$ into the set of admissible colour $Co(p)$ and each transition $t \in T$ into the set of admissible fireable colour $Co(t)$. Let $\Omega$ defined as: $\Omega = \bigcup x \in P \cup T\{Co(x)\}$.

• The colour of each token $j$ is a couple: $\langle \lambda j, \theta j \rangle$, where $\lambda j$ is the batch type and $\theta j$ the corresponding assigned quantity $\theta j$.

• The marking $M$ denotes the state of the CTPN: $M(p)$ gives the colours associated with each token in $p$. $MO$ is the initial marking.
Model with CTPN (V): PN Definition

- \( \text{CTPN} = (P, T, Co, Pre, Post, \Omega, F, RS, M_0) \).

- \( F \): specifies the firing time associated to each transition (exponential distribution or null value).

- Matrix \( Pre \) gives the enabling condition:
  \( t \in T \) is enabled at marking \( M \) with respect to a colour \( c \in Co(t) \) iff for each \( p \) in input of \( t \), it holds
  \( M(p) \geq Pre(p, t)(c) \).

- Matrices \( Pre \) and \( Post \) update the marking: upon firing, \( t \) leads to a new marking \( M' \):
  \( M'(p) = M(p) + Post(p, t)(c) - Pre(p, t)(c) \).
The model of a general purpose site (manufacturer/assembler, distributor, de-manufacturer, supplier)

Inventory control rule: \((R, Q)\)

- \(p_i\): the product is in the input buffer
- \(p_{ic}'\): the available capacity.
- \(t_r\): the transport of material.
- \(t_I\): the uploading in the production buffer.
- \(p_P\): the product is in production
- \(p_{Pc}'\): the capacity of the production facility.
- \(t_p\): the production operation
- \(p_o\): the product is in the output buffer
- \(t_o\): the transport uploading.
The input buffer can accomodate $n_1$ lots of type $\lambda_1$ and of $\theta_1$ pieces.

Capacities, reorder functions and reorder quantities can be different for each type of product:

$$\text{Cap}(p_{I_c}) = (n_1\langle\lambda_1,\theta_1\rangle, n_2\langle\lambda_2,\theta_2\rangle, \ldots)$$
The model of a general purpose site (manufacturer/assembler, distributor, de-manufacturer, supplier)

UP is a function that updates the token colors with the new color obtained after the operation.

No label stands for “the function makes no transformation in the elements”.
The model of a general purpose site (manufacturer/assembler, distributor, de-manufacturer, supplier)

The output inventory of product $\lambda 1$ in $p_O$ is higher than the output reorder point, i.e. $M(p_O) > R(p_{Oc})(<\lambda 1, \theta 1>)$: Transition $t_I$ is inhibited but $t_P$ continue to produce.

The output inventory of product $\lambda 1$ in $p_O$ drops below the output reorder point, i.e. $M(p_O) < R(p_{Oc})(<\lambda 1, \theta 1>)$: Transition $t_I$ is enabled with respect to the color $<\lambda 1, \theta 1>$: $Q(p_{Oc})(<\lambda 1, \theta 1>)$ products of type $\lambda 1$ will be produced.
The CTPN modelling the SC sites (V)

The model of a transporter
- $p_T$: batch under transport
- $p_{Tc}$: transport capacity
- $t_T$: transport operation
- $t_O$: the transport uploading

The model of a customer
- $p_U$: available products
- $p_D$: outstanding orders
- $t_{oD}$: arrival of customer demand
- $t_o$: marketing of product of a recycler
Case Study B (I):

- Two connections from suppliers to retailers are added
- Different products are added

3 suppliers

2 manufacturers

1 distributor

2 retailers

1 de-manufacturer
Case Study B (II): CTPN model

3 suppliers

2 manufacturers

1 distributor

2 retailers

1 de-manufacturer

The model is compact and is able to distinguish among different types of products
Case Study B (III): Performance indices

- **T = Throughput** The average number of products per time unit

- **Inventory = INin + INout** Inventory level of buffers

- **INin** Average number of products in the input buffers

- **INout** Average number of products in the output buffers

- **CTij = Order fulfillment cycle time of the product j-th for the i-th customer**, i.e. the average time elapsing from the i-th customer request and the j-th product delivery.

- **CT = Order fulfillment cycle time**, i.e. the average time elapsing from a customer request and the product delivery.
## Case Study B (IV): Data

### Data buffers and transporters

### Average firing time (hours) of exponential transitions of CTPN

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<td>$s_5$</td>
<td>$s_6$</td>
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<td>-</td>
<td>2100</td>
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<td>M ($\lambda_4$)</td>
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<td>-</td>
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<td>K ($\lambda_5$)</td>
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| Capacità dei magazzini [dm$^3$] | 100000 |
| Capacità dei trasportatori [dm$^3$] | 600000 | 900000 |

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<td>0.83</td>
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<td>0.02</td>
<td>3.33</td>
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Case Study B (V): Simulation results

- The CTPN is simulated in the ARENA environment
- Places model resource with limited capacity, timed transitions model delay, tokens model entities and colors model attributes of entities.

20 independent replications of TP=11856 h, with a transient period of TR=3025 h. The half width of the confidence interval, being 2.7% in the worst case.

Since customers have the same order frequency, throughputs are comparable.

M. Dotoli, Automatica.it, Pisa, 7-9 settembre 2011
Case Study B (VI): Simulation results

- The CT42 and CT52 are lower for the lower values of the firing times.
- The input buffers tend to accommodate more parts than the output ones.
- The CT are about 7h consistent with the firing times.

M. Dotoli, Automatica.it, Pisa, 7-9 settembre 2011
• Introduction to Supply Chains (SC)
• Aim of the research
• SC operational model by First Order Hybrid Petri Nets (FOHPN)
• SC operational model by Coloured Timed Petri Nets (CTPN)
• Conclusions and Further Research
Conclusions and Future Research

We represent SC using the PN discrete event systems formalism.

We propose two alternative SC modular models, devoted to the efficient description, management, and performance evaluation of SC at the operational level.

The FOHPN model

- describes material as continuous flows and the stochastic events occurring in the system as discrete events;
- is resource oriented, modular and efficient;
- enables the designer to impose an optimal SC dynamics according to objective functions and the knowledge of the system state.

The CTPN model

- describes in detail the SC products flow;
- is resource oriented, modular and efficient;
- is suitable for implementing control strategies.

Future research: a hierarchical decision support system based on the two modelling frameworks for SC management at the operational level.
Petri Net Models for Supply Chain Operational Management and Performance Evaluation

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